Notation.
For a positive integer x let $\Omega(x)$ be the number of prime factors of x counted with multiplicity ($\Omega(1) = 0$).
For a prime p let $v_p(x)$ be the exponent of p in the factorisation of x ($v_p(1) = 0$).

The numbers on the blackboard at any moment form a multiset $\mathcal{S}$ of size 2026.
Define

$$N(\mathcal{S}) = \bigl|\{x \in \mathcal{S} : x > 1\}\bigr|, \qquad T(\mathcal{S}) = \sum_{x \in \mathcal{S}} \Omega(x).$$

Both N and T are non-negative integers.

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1. Effect of one move on N and T

Pick two numbers $a, b \in \mathcal{S}$ with $a, b > 1$ and replace them by

$$g = \gcd(a, b), \qquad h = \frac{\operatorname{lcm}(a, b)}{\gcd(a, b)}.$$

Both g and h are positive integers.
Because $ab = \operatorname{lcm}(a, b) \cdot g$ and Ω is completely additive,

$$\Omega(a) + \Omega(b) = \Omega(\operatorname{lcm}(a, b)) + \Omega(g). \tag{1}$$

For the new pair,

$$\Omega(g) + \Omega(h) = \Omega(g) + \Omega\Bigl(\frac{\operatorname{lcm}(a, b)}{g}\Bigr).$$

Since $\operatorname{lcm}(a, b) = g \cdot \frac{\operatorname{lcm}(a,b)}{g}$,

$$\Omega(\operatorname{lcm}(a, b)) = \Omega(g) + \Omega\Bigl(\frac{\operatorname{lcm}(a, b)}{g}\Bigr),$$

hence

$$\Omega(g) + \Omega(h) = \Omega(\operatorname{lcm}(a, b)). \tag{2}$$

Comparing (1) and (2) we see that the contribution of the two chosen numbers to T changes from $\Omega(a) + \Omega(b)$ to $\Omega(\operatorname{lcm}(a, b))$; therefore T **decreases** by $\Omega(g)$.
In particular T never increases, and it strictly decreases whenever $g > 1$ (because then $\Omega(g) \geq 1$).

Now examine N. We remove the two entries $a, b > 1$ and insert g, h.

- If $g = 1$, then $h = \operatorname{lcm}(a,b) = ab > 1$. We obtain one 1 and one number > 1; hence N drops by 1.
- If $g > 1$, then $g > 1$. Write $h = \frac{a}{g} \cdot \frac{b}{g}$ (the two factors are coprime).
  – If $a = b$ then $h = 1$; again we get one > 1 and one 1, so N drops by 1.
  – If $a \neq b$ then the two factors cannot both be 1; therefore $h > 1$. Both g and h are > 1, so N stays unchanged.

In every move **either** N decreases by 1 **or** N remains the same and $g > 1$ (which forces T to decrease by at least 1).
Consequently the ordered pair (N, T) strictly decreases in the lexicographic order:

$$(N_{\text{new}}, T_{\text{new}}) < (N_{\text{old}}, T_{\text{old}}).$$

Both N and T are non-negative integers. N never increases, so it can drop at most 2026 times. When N stays constant, T strictly decreases by at least 1; moreover T never increases in any move. Hence the total number of moves is bounded by $2026 + T_{\text{initial}}$ – the process must terminate after finitely many moves.
When it stops, no two numbers > 1 are available, so $N \leq 1$.

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2. An invariant

Fix a prime p. For a state $\mathcal{S}$ set

$$d_p(\mathcal{S}) = \gcd\{\, v_p(x) : x \in \mathcal{S} \,\}.$$

(If all valuations are 0, the gcd is 0; otherwise it is a positive integer.)

Take a move on a, b and let $a_p = v_p(a),\ b_p = v_p(b)$. After the move the two numbers carry the exponents $\min(a_p, b_p)$ and $|a_p - b_p|$, while the other 2024 numbers keep their exponents.
For any non-negative integers u, v,

$$\gcd(u,v) = \gcd(\min(u,v),\ |u - v|)$$

(if $u \geq v$ then $\min = v,\ |u - v| = u - v$; any common divisor of u, v divides both v and $u - v$, and conversely). Hence the gcd of the two new exponents equals $\gcd(a_p, b_p)$. The remaining 2024 exponents are untouched, so the gcd of the whole multiset of p-adic valuations does not change.

Thus $d_p(\mathcal{S})$ **is invariant** during the entire process.

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3. At least one number always stays > 1

Initially all 2026 numbers are > 1, so their product is > 1. Hence there exists a prime p that occurs in at least one initial number. For this p the initial multiset of exponents contains a positive integer, therefore its gcd d_p is ≥ 1. By invariance, in every reachable state we still have $d_p \geq 1$.

If all numbers on the board were 1, every v_p would be 0 and the gcd would be 0, a contradiction. Thus in every state there is at least one number with $v_p(x) \geq 1$, i.e. $x > 1$. Consequently $N \geq 1$ **always**.

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4. Termination – exactly one > 1 remains

We already know that the process terminates with $N \leq 1$. Together with $N \geq 1$ we obtain $N = 1$.
Hence exactly one integer $M > 1$ remains on the blackboard, while the other 2025 entries are 1. This proves **part (a)**.

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5. The value of M is independent of the choices

In the terminal state the multiset of p-adic valuations is

$$\{\, v_p(M),\ 0,\ 0,\ \dots,\ 0 \,\} \qquad \text{(2025 zeros)}.$$

The gcd of this set equals $v_p(M)$ (because $\gcd(x, 0) = x$). By the invariance of d_p we must have

$$v_p(M) = d_p \qquad \text{for every prime } p,$$

where d_p is the invariant computed from the initial configuration. Therefore

$$M = \prod_p p^{d_p}$$

(only finitely many primes have $d_p > 0$, so the product is a well-defined integer).
The right-hand side depends only on the 2026 original numbers, not on the sequence of moves. Hence the final number M is uniquely determined and does not depend on Confucius' choices. This proves **part (b)**.
